ΘΕΜΑ Α

$A_1 \longrightarrow$ β

$A_2 \longrightarrow$ γ

$A_3 \longrightarrow$ α

$A_4 \longrightarrow$ γ

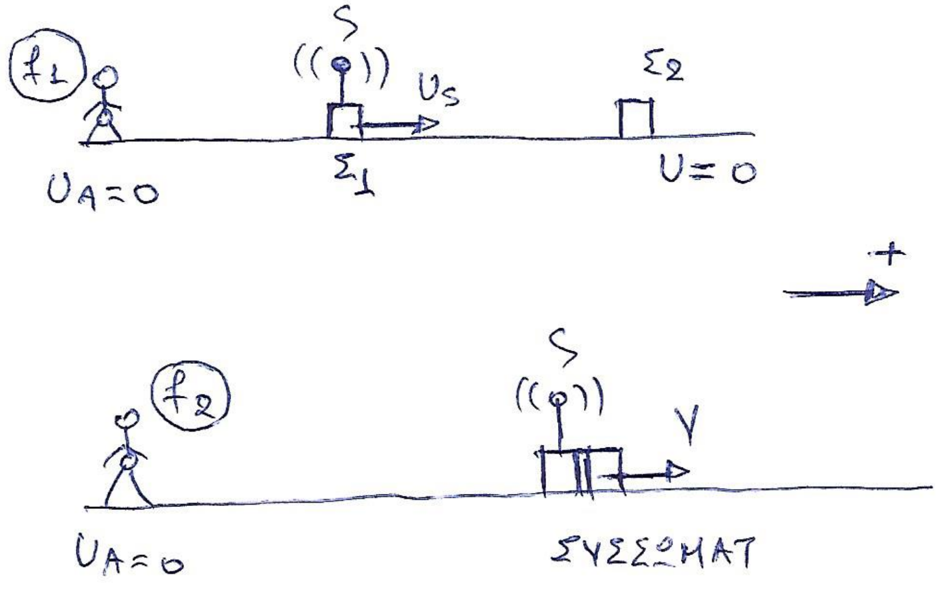
$A_5 \longrightarrow$ Λ, Σ, Λ, Σ, Σ

ΘΕΜΑ Β

B1 σωστό (ii)

ΠΡΙΝ ΚΡΟΥΣΗ

ΜΕΤΑ ΚΡΟΥΣΗ



$$\vec{P}_{πριν} = \vec{P}_{μετ} \Rightarrow m U_s + m \cdot 0 = (m+m)V \Rightarrow \boxed{V = \frac{U_s}{2}}$$

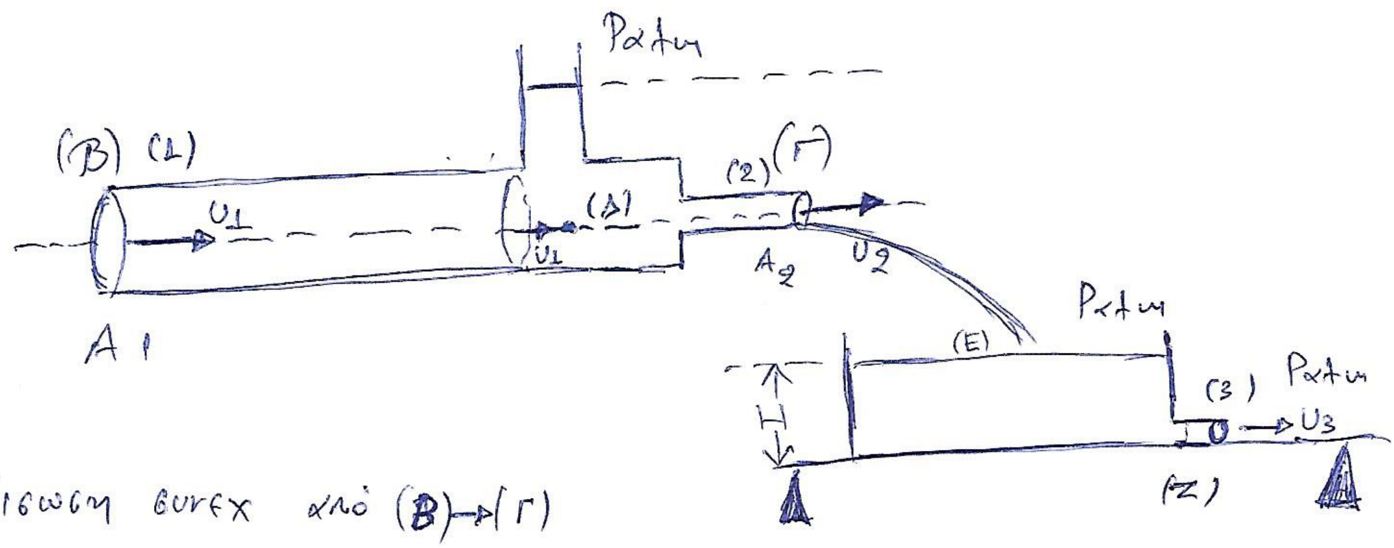
$$V = \frac{U_s}{2} \Rightarrow V = \frac{U_H/20}{2} \Rightarrow V = \frac{U_H}{40}$$

$$f_1 = \frac{U_{\eta\chi} + 0}{U_{\eta\chi} + U_s} f_s \quad \text{και} \quad f_2 = \frac{U_{\eta\chi} + 0}{U_{\eta\chi} + V} f_s$$

$$\frac{f_1}{f_2} = \frac{\dfrac{\cancel{U_{\eta\chi}}}{U_{\eta\chi}+U_s} \cancel{f_s}}{\dfrac{\cancel{U_{\eta\chi}}}{U_{\eta\chi}+V} \cancel{f_s}} \Rightarrow \frac{f_1}{f_2} = \frac{U_{\eta\chi}+V}{U_{\eta\chi}+U_s} \Rightarrow$$

$$\frac{f_1}{f_2} = \frac{U_{\eta\chi} + \dfrac{U_{\eta\chi}}{40}}{U_{\eta\chi} + \dfrac{U_{\eta\chi}}{20}} \Rightarrow \frac{f_1}{f_2} = \frac{\dfrac{41}{40}\cancel{U_{\eta\chi}}}{\dfrac{21}{20}\cancel{U_{\eta\chi}}} \Rightarrow \boxed{\frac{f_1}{f_2} = \frac{41}{42}}$$

Β2 σωστό (iii)



Εξίσωση συνεχ. από (B)→(Γ)

$$\Pi_1 = \Pi_2 \Rightarrow A_1 \cdot U_1 = A_2 \cdot U_2 \Rightarrow 2A_2 U_1 = A_2 U_2 \Rightarrow$$

$$U_2 = 2U_1 \quad (1)$$

$$\left.\begin{array}{l} P_\Delta + \frac{1}{2}\rho U_1^2 = P_2 + \frac{1}{2}\rho U_2^2 \\ P_\Delta = P_{atm} + \rho g h \end{array}\right\} \Rightarrow \cancel{P_{atm}} + \rho g h + \frac{1}{2}\rho U_1^2 = \cancel{P_{atm}} + \frac{1}{2}\rho U_2^2$$

$$gh + \frac{1}{2}U_1^2 = \frac{1}{2}U_2^2 \overset{1}{\Rightarrow} gh + \frac{1}{2}\frac{U_1^2}{4} = \frac{1}{2}U_2^2 \Rightarrow$$

$$gh = \frac{3}{8}U_2^2 \Rightarrow U_2 = \sqrt{\frac{8gh}{3}} \quad (2)$$

Στό δοχείο αφού επιφάνεια σταθερή

$$\Pi_2 = \Pi_3 \Rightarrow A_2 U_2 = A_3 U_3 \Rightarrow$$

$$A_2 U_2 = \frac{A_2}{2} U_3 \Rightarrow U_3 = 2U_2 \quad (3)$$

Bernoulli (E) → (Z):

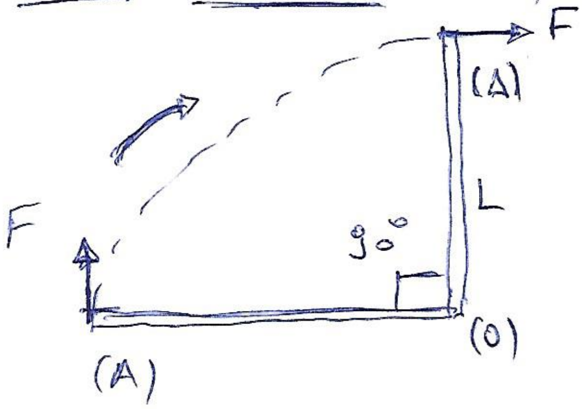
$$\cancel{P_{atm}} + \cancel{\rho} g H + 0 = \cancel{P_{atm}} + \frac{1}{2}\cancel{\rho} U_3^2 \overset{3}{\Rightarrow}$$

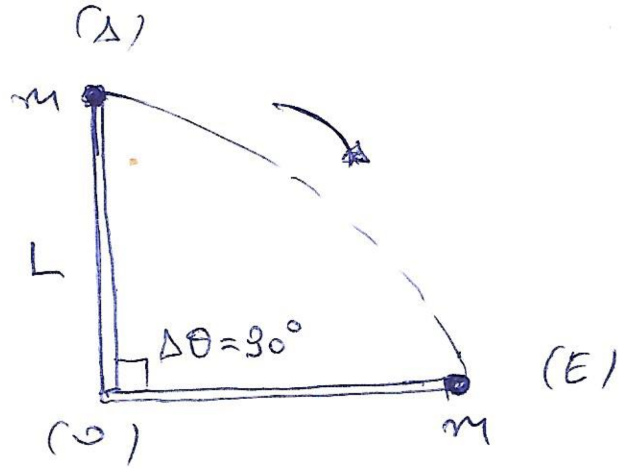
$$gH = \frac{1}{2} 4U_2^2 \Rightarrow gH = 2U_2^2 \Rightarrow U_2 = \sqrt{\frac{gH}{2}} \quad (4)$$

Από (2) και (4) $\sqrt{\frac{8}{3}gh} = \sqrt{gH/2} \Rightarrow \frac{8}{3}\cancel{g}h = \cancel{g}\frac{H}{2} \Rightarrow$

$$\boxed{\frac{h}{H} = \frac{3}{16}}$$

Β3 | σωστό (ii)





κίνηση (Α) → (Δ):

$$K_{τελ} - K_{αρχ} = W_F \Rightarrow \frac{1}{2} I\omega^2 = F\cdot L\cdot \frac{\pi}{2} \Rightarrow$$

$$\frac{1}{\cancel{2}}\frac{1}{3} M L^{\cancel{2}} \omega^2 = F\cdot \cancel{L}\cdot \pi/\cancel{2} \Rightarrow \frac{1}{3} M L\cdot\omega^2 = F\cdot\pi \Rightarrow$$

$$\omega = \sqrt{\frac{3F\cdot\pi}{M\cdot L}} \Rightarrow \omega = \sqrt{\frac{\cancel{3}\cdot 9\pi\cdot\pi}{\cancel{3}\cdot 1}} \Rightarrow \omega = 3\pi \text{ rad/s}$$

Α.Δ.ΣΤΡ. στο (Δ): $\vec{L}_{πριν} = \vec{L}_{μετά} \Rightarrow I\omega = I'\omega' \Rightarrow$

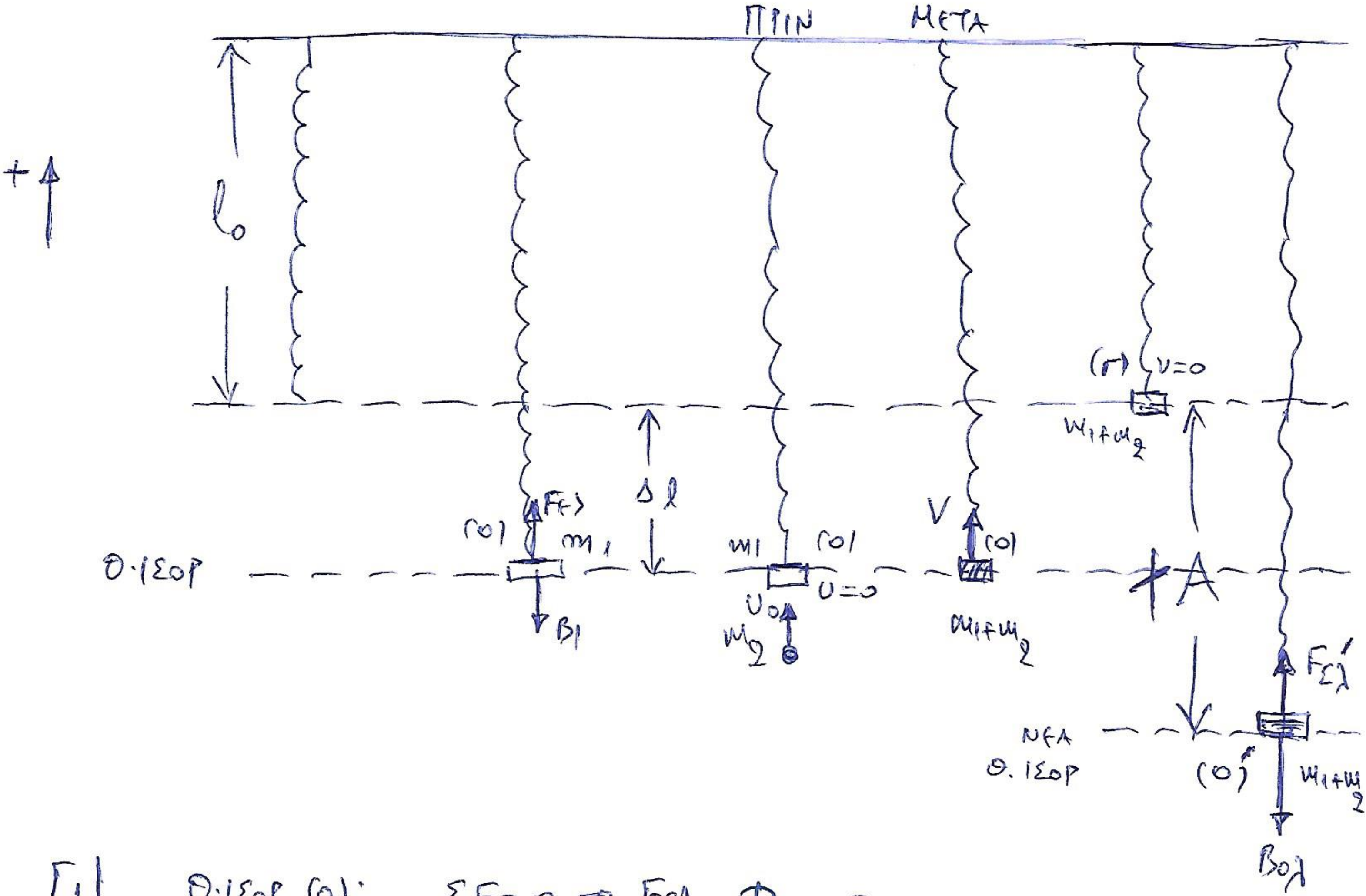
$$\frac{1}{3} M L^2 \omega = \left(\frac{1}{3} M L^2 + m L^2\right)\cdot\omega' \Rightarrow$$

...... $\boxed{\omega' = 3\pi/2}$ rad/s

$$\Delta\theta = \omega' t \Rightarrow \frac{\pi}{2} = \frac{3\pi}{2} t \Rightarrow \boxed{t = \frac{1}{3} \text{ sec}}$$

ΘΕΜΑ 3°

K, $m_1 = 1Kgr$, $\Delta l = 0{,}05m$, $m_2 = 1Kgr$, U_0



Γ1. Θ.ΙΣΟΡ (0): $\Sigma F = 0 \Rightarrow F_{ελ} = B_1 \Rightarrow$

$K \cdot \Delta l = m_1 \cdot g \Rightarrow K \cdot 0{,}05 = 10 \Rightarrow K = 200 N/m$

Α.Δ.Ο $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow m_2 \cdot U_0 + m_1 \cdot 0 = (m_1+m_2) V \xRightarrow{m_1 = m_2 = m}$

$\cancel{m} U_0 = 2\cancel{m} V \Rightarrow \boxed{V = \frac{U_0}{2}}$ (1)

ΝΕΑ Θ. ΙΣΟΡ (0'): $\Sigma F' = 0 \Rightarrow F_{ελ}' = B_{ολ} \Rightarrow K \cdot A = m_{ολ} \cdot g \Rightarrow$

$200 \cdot A = 20 \Rightarrow \boxed{A = 0{,}1m}$

A.Δ.Ε.Τ $E_{ολ(0)} = E_{ολ(Γ)} \Rightarrow$

$$K_{(0)} + U^{ταλ}_{(0)} = \cancel{K_{(Γ)}}^{0} + U^{ταλ}_{(Γ)} \Rightarrow$$

$$\frac{1}{2} m_{ολ} V^2 + \frac{1}{2} K(A-\Delta \ell)^2 = \frac{1}{2} K A^2 \Rightarrow$$

$$2 \cdot V^2 + 200(0{,}1-0{,}05)^2 = 200 \cdot 0{,}1^2 \Rightarrow$$

$$V = \sqrt{3}/2 \ m/sec$$

από ① $V = \frac{U_0}{2} \Rightarrow \frac{\sqrt{3}}{2} = \frac{U_0}{2} \Rightarrow U_0 = \sqrt{3}\ m/s$

Γ2 $K_2 = \frac{1}{2} m_2 U_0^2 = \frac{1}{2} \cdot 1 \, (\sqrt{3})^2 = 1{,}5\ J$

Γ3 $\Delta \vec{P}_2 = \vec{P}_2' - \vec{P}_2 \Rightarrow \Delta P_2 = m_2 V - m_2 U_0 \Rightarrow$

$\Delta P_2 = \frac{\sqrt{3}}{2} - \sqrt{3} \Rightarrow \Delta P = -\frac{\sqrt{3}}{2}\ Kgr\ m/sec$ φορά προς τα κάτω

Γ4 $X = A\eta\mu(\omega t + \phi_0) \xRightarrow[x = \Delta \ell = +0{,}05m]{t=0}$

$0{,}05 = 0{,}1\ \eta\mu\phi_0 \Rightarrow \eta\mu\phi_0 = 1/2 = \eta\mu\ \pi/6$

$\phi_0 = 2\kappa\pi + \pi/6$ | $\phi_0 = \pi/6$

$\phi_0 = 2\kappa\pi + \pi - \pi/6$ | $\phi_0 = 5\pi/6$

και επειδή $V > 0$ $\boxed{\phi_0 = \pi/6}$ ......

$$\omega = \sqrt{\frac{K}{m_{ολ}}} = \sqrt{\frac{200}{2}} = 10\ rad/s$$

Άρα $$\boxed{X = 0{,}1\ \eta\mu\left(10t + \frac{\pi}{6}\right)}$$

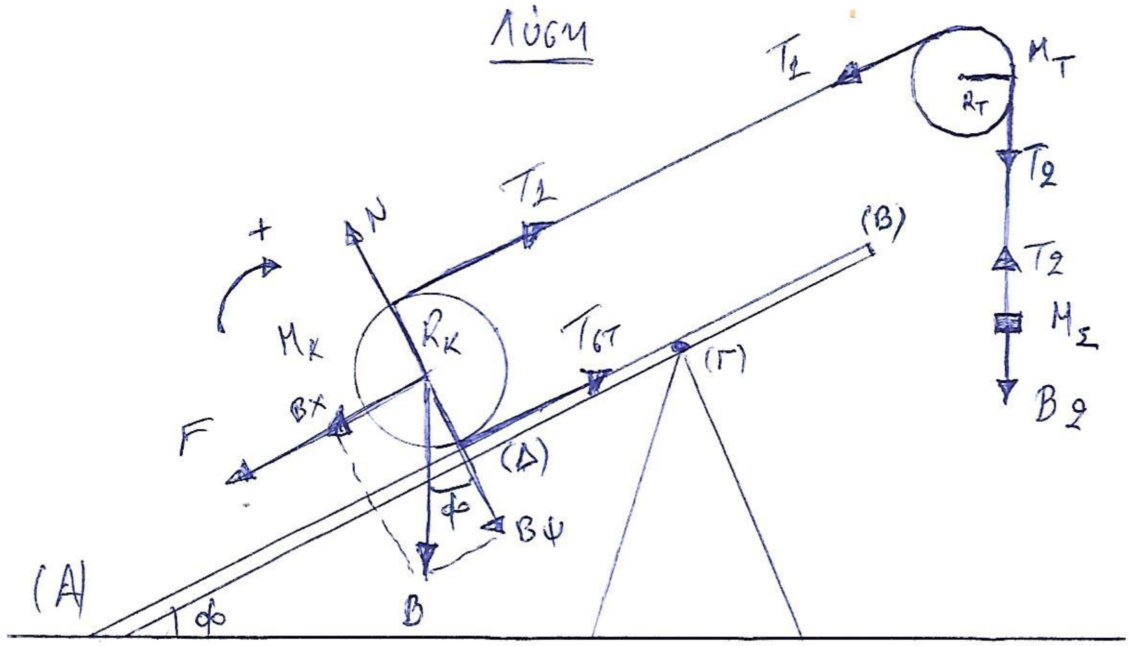
ΘΕΜΑ 4ο

$(AB) = \ell = 4\,m$, $M = 2\,Kgr$, $\phi = 30^\circ$, $(B\Gamma) = 1,5\,m$

$M_K = 2\,Kgr$, R_K, $(\Gamma\Delta) = 0,2\,m$, $M_\Sigma = 2\,Kgr$, $M_T = 2\,Kgr$, R_T

Λύση

Δ1



κύλινδρ: $B_x = B\,\eta\mu\phi \Rightarrow B_x = M_K \cdot g\,\eta\mu 30^\circ \Rightarrow B_x = 2 \cdot 10 \cdot \frac{1}{2} \Rightarrow B_x = 10N$

$\Sigma F_x = 0 \Rightarrow T_1 + T_{στ} - F - B_x = 0 \Rightarrow$

$T_1 + T_{στ} = F + 10$ (1)

$\Sigma \tau = 0 \Rightarrow T_1 \cdot R_K - T_{στ} \cdot R_K = 0 \Rightarrow T_1 = T_{στ}$ (2)

τροχαλία: $\Sigma \tau' = 0 \Rightarrow T_2 \cancel{R_T} - T_1 \cancel{R_T} = 0 \Rightarrow T_2 = T_1$ (3)

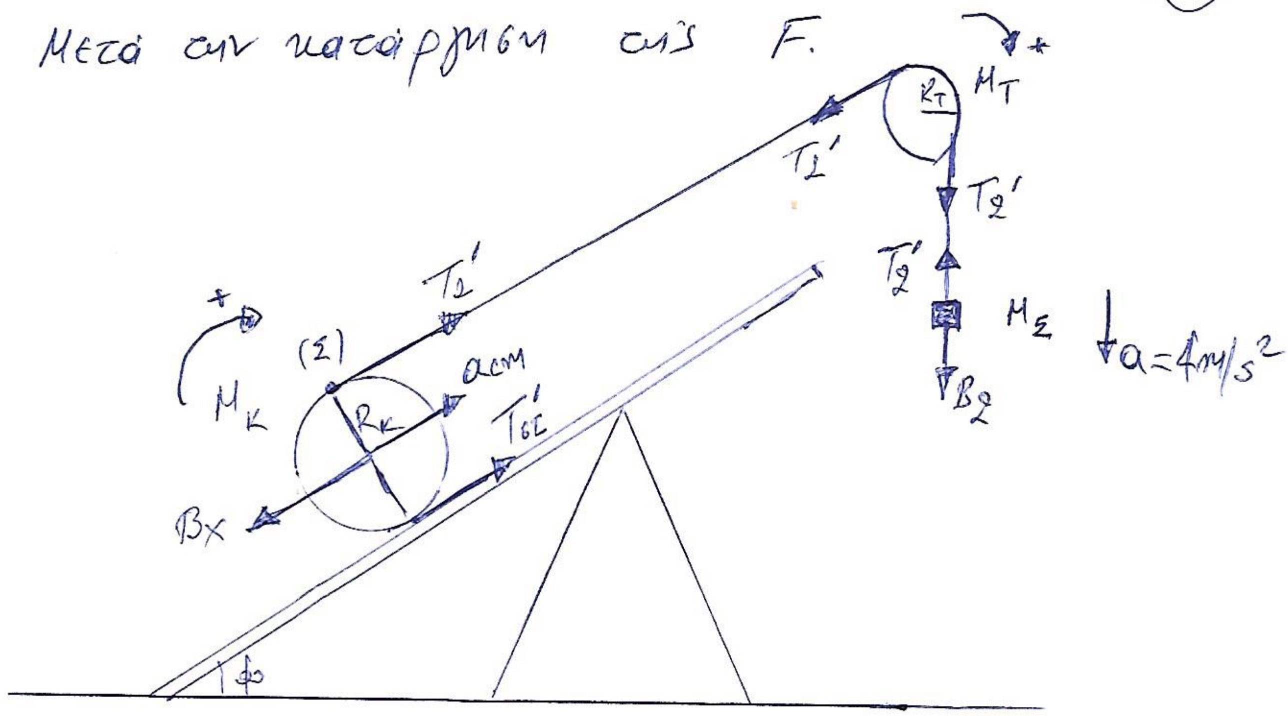
σώμα Σ: $\Sigma F_\psi = 0 \Rightarrow B_2 = T_2 \Rightarrow M_\Sigma \cdot g = T_2 \Rightarrow T_2 = 20N$

άρα από (3) $T_1 = T_2 = 20N$

από (2) $T_1 = T_{στ} \Rightarrow T_{στ} = 20N$

(1) $20 + 20 = F + 10 \Rightarrow \boxed{F = 30N}$

Δ2 Μετά την κατάργηση της F.



για το σημείο (Σ) του κυλίνδρου ισχύει:

$$\left.\begin{array}{l} a_Σ = a \\ a_Σ = a_{cm} + a_{επιτρ} \Rightarrow a_Σ = 2a_{cm} \\ (a_{cm} = a_{επιτρ}) \end{array}\right\} \quad 2a_{cm} = a \Rightarrow \boxed{a_{cm} = \frac{a}{2}}$$

$M_Σ$: $ΣF_y = M_Σ \cdot a \rightarrow B_2 - T_2' = M_Σ \cdot a \Rightarrow \underline{20 - T_2' = 2 \cdot a}$ (1)

M_T: $Στ = I_T\, α_{γων} \Rightarrow T_2' \cancel{R_T} - T_1' \cancel{R_T} = \frac{1}{2} M_T R_T^2 \cdot α_{γων} \xRightarrow{a = α_{γων} R_T}$

$$T_2' - T_1' = \frac{1}{2} \cdot 2 \cdot a \Rightarrow \underline{T_2' - T_1' = a} \quad (2)$$

κυλινδρ M_K: $ΣF_x = M_K \cdot a_{cm} \Rightarrow T_1' + T_{στ}' - B_x = M_K \cdot a_{cm} \Rightarrow$

$$T_1' + T_{στ}' - 10 = \cancel{2} \cdot \frac{a}{\cancel{2}} \Rightarrow \underline{T_1' - T_{στ}' - 10 = a} \quad (3)$$

$$Στ' = I_K \cdot α_{γων}' \Rightarrow T_1' \cancel{R_K} - T_{στ}' \cancel{R_K} = \frac{1}{2} M_K R_K^{\cancel{2}}\, α_{γων}' \xRightarrow{a_{cm} = α_{γων}' R_K}$$

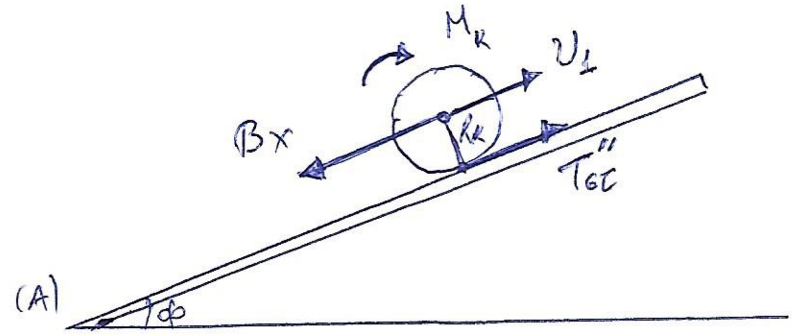
$$T_1' - T_{στ}' = \frac{1}{\cancel{2}} \cdot \cancel{2}\, a_{cm} \Rightarrow \underline{T_1' - T_{στ}' = \frac{a}{2}} \quad (4)$$

Λύνοντας το σύστημα των εξισώσεων

1, 2, 3, 4 (ΘΕΛΕΙ ΠΡΟΣΟΧΗ !!!) έχουμε

$\boxed{a = 4 m/sec^2}$ και $a_{cm} = \frac{a}{2} = 2 m/sec^2$

Δ3)



όταν κοπεί το νήμα την $t_1 = 0,5$ sec ο κύλινδρος είχε

ταχύτητα: $U_1 = a_{cm} \cdot t_1 \Rightarrow U_1 = 2 \cdot 0,5 = 1 m/sec$

και πλέον "περνάει" σε επιβραδυνόμενη κίνηση:

$\Sigma F_x = M_K a'_{cm} \Rightarrow B_x - T_{στ}'' = M_K \cdot a'_{cm} \Rightarrow$

$\underline{10 - T_{στ}'' = 2 a'_{cm}}$ (1) (a'_{cm}: η νέα επιβράδυνση)

$\Sigma \tau = I_K \cdot a_{γων}'' \Rightarrow T_{στ}'' \cdot \cancel{R_K} = \frac{1}{2} M_K R_K^{\cancel{2}} \cdot a_{γων}'' \Rightarrow T_{στ}'' = \frac{1}{2} \cdot 2 \cdot a'_{cm} \Rightarrow$

$\underline{T_{στ}'' = a'_{cm}}$ (2) (όπου $a'_{cm} = a_{γων}'' R_K$)

(1) + (2) $10 - \cancel{T_{στ}''} + \cancel{T_{στ}''} = 2a'_{cm} + a'_{cm} \Rightarrow 10 = 3a'_{cm} \Rightarrow$

$a'_{cm} = 10/3 \ (m/sec^2)$ (επιβράδυνση)

$U_{τελ} = U_1 - a'_{cm} \Delta t \Rightarrow 0 = 1 - \frac{10}{3} \Delta t \Rightarrow \Delta t = 0,3 sec$

Άρα $t_2 = t_1 + \Delta t \Rightarrow t_2 = 0,5 + 0,3 = 0,8$ sec επιταχυνόμενη ομαλή κίνηση:

Δ4]. $S_1 = \frac{1}{2} a_{\varepsilon\pi} t_1^2 = \frac{1}{2} \cdot 2 \cdot 0{,}5^2 = 0{,}25\,m$

επιβραδυνόμενη ομαλή κίνηση:

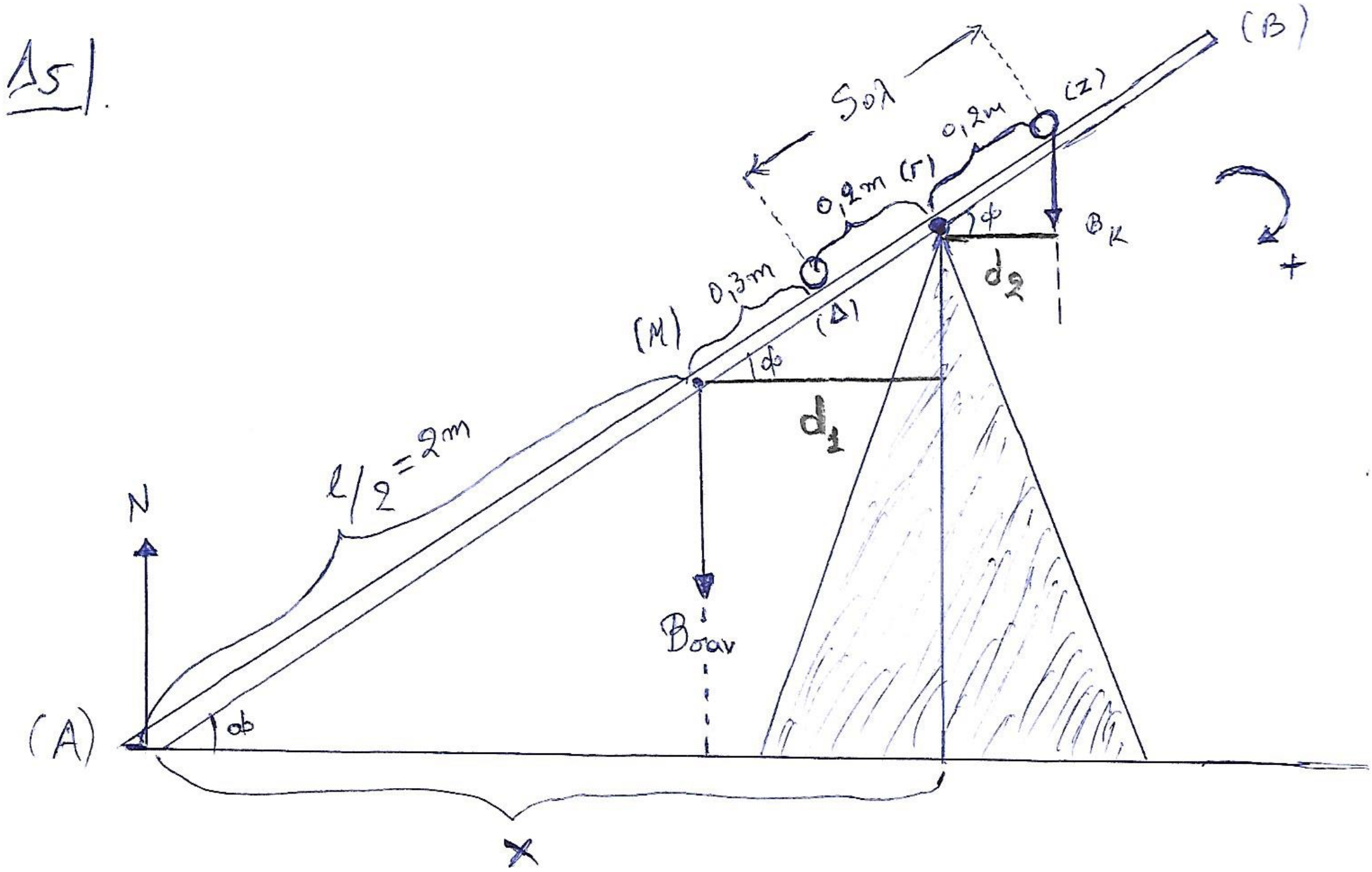
$S_2 = U_1 \cdot \Delta t - \frac{1}{2} a_{\varepsilon\pi} \Delta t^2 \Rightarrow$

$S_2 = 1 \cdot 0{,}3 - \frac{1}{2} \frac{10}{3} 0{,}3^2 \to$

$S_2 = 0{,}3 - \frac{0{,}9}{6} \Rightarrow S_2 = 0{,}15\,m$

Άρα $S_{o\lambda} = S_1 + S_2 = 0{,}25 + 0{,}15 = \underline{0{,}4\,m}$

Δ5].



Η σανίδα θα ανατραπεί μόνο αν ο κύλινδρος "περάσει" πάνω από το σημείο (Γ) της άρθρωσης του υποστηρίγματος. Από το $S_{ολ} = 0{,}4m$ που βρήκαμε στο προηγούμενο ερώτημα ο κύλινδρος μπορεί να φτάσει μέχρι το (Ζ) του προηγούμενου σχήματος.

Υπολογίζω την κάθετη αντίδραση N που δέχεται η σανίδα:

$$\Sigma\tau^{\Gamma} = 0 \Rightarrow \tau_N^{\Gamma} + \tau_{B_{σαν}}^{\Gamma} + \tau_{B_K}^{\Gamma} = 0 \rightarrow$$

$$\left.\begin{array}{l} N\cdot X - B_{σαν}\, d_1 + B_K\, d_2 = 0 \\ συνφ = \dfrac{d_2}{(ΓΖ)} \Rightarrow d_2 = 0{,}2\cdot συνφ \\ συνφ = \dfrac{d_1}{(ΜΓ)} \Rightarrow d_1 = 0{,}5\cdot συνφ \\ συνφ = \dfrac{X}{(ΑΓ)} \Rightarrow X = 2{,}5\cdot συνφ \end{array}\right\} \begin{array}{l} B_{σαν} = Mg = 20N \\ \Rightarrow \\ B_K = M_K\cdot g = 20N \end{array}$$

$$N\cdot 2{,}5συνφ - 20\, 0{,}5\cdot συνφ + 20\, 0{,}2\cdot συνφ = 0$$

$$N\cdot 2{,}5\cancel{συνφ} = 10\cancel{συνφ} - 4\cancel{συνφ} \Rightarrow$$

$$N\cdot 2{,}5 = 6 \Rightarrow \boxed{N = 2{,}4 \text{ Newton}}$$

Άρα αφού $N \neq 0$ η σανίδα δεν ανατρέπεται.

ΜΑΚΗΣ ΙΟΡΔΑΝΙΔΗΣ